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Confronting hybrid inflation in supergravity with CMB data

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Abstract: F-term GUT inflation coupled to $N = 1$ supergravity is confronted with CMB data. Corrections to the string mass-per-unit-length away from the Bogomolny limit are taken into account. We find that a superpotential coupling $10^{-7}/\mathcal{N} \lesssim \kappa \lesssim 10^{-2}/\mathcal{N}$, with $\mathcal{N}$ the dimension of the Higgs-representation, is still compatible with the data. The parameter space is enlarged in warm inflation, as well as in the curvaton and inhomogeneous reheat scenario. F-strings formed at the end of P-term inflation are also considered. Because these strings satisfy the Bogomolny bound the bounds are stronger: the gauge coupling is constrained to the range $10^{-7} < g < 10^{-4}$.

Keywords: Solitons Monopoles and Instantons, Supersymmetry Phenomenology, Cosmology of Theories beyond the SM, GUT.

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1. Introduction

The cosmic microwave background (CMB) power spectrum measured by WMAP points to a predominantly adiabatic perturbation spectrum, as produced in standard inflation [1]. The existence of the acoustic peaks excludes cosmic strings as the main source of perturbations, although a 10% contribution is not excluded [2]. This has important implications for hybrid inflation [3] because almost all particle physics models of hybrid inflation, such as standard SUSY GUT F-term inflation, D-term inflation and brane inflation, predict the formation of cosmic strings at the end of inflation [4 − 6]. The string contribution to the CMB anisotropies in D-term models is far too high [4] unless the gauge coupling constant is unnaturally small [7]. In this paper we focus on F-term inflation which can naturally arise from SUSY GUTs, and as a low energy effective description of a certain class of interacting D-brane models.

The minimal field content of F-term inflation is a gauge singlet superfield and two Higgs superfields which transform in complex conjugate representations of some gauge group G. Inflation takes place as the singlet field slowly rolls down a valley of local minima where the vacuum expectation value (VEV) of the Higgs fields vanish and G is unbroken. When the scalar singlet falls below a certain critical value, the Higgs mass becomes tachyonic and inflation ends very rapidly, via a phase transition during which the Higgs fields acquire a non-vanishing VEV and G spontaneously breaks down to a subgroup H. Of all topological defects that can form during such a phase transition, only cosmic strings are cosmologically viable.

In the context of GUTs, where monopoles ought to form, this implies that monopoles should be formed in a phase transition prior to inflation. Hence, the group G which gets broken at the end of inflation is not the GUT group itself, but some intermediate symmetry group. There will be more phase transitions after inflation if H is larger than the standard model gauge group, and these should not lead to the formation of unwanted defects. These arguments together with the observation that the rank of the symmetry group G which is broken at the end of inflation is reduced by one unit ($\mathrm{rank}(G) = \mathrm{rank}(H) + 1$) and simple homotopy arguments, lead to the conjecture that cosmic strings always form at the end of GUT hybrid inflation [4]. This was proven for an SO(10) GUT [8], and recently for all GUT groups with rank less than 8 [5]. The strings which form at the end of inflation are topological and remain stable down to low energies if R-parity remains unbroken.

The constraints from the CMB data can be avoided if the cosmic strings are unstable, or if no strings are formed at all. We note that this only happens in rather specific models. The embedded strings arising in GUT models in which R-parity is broken can be unstable, depending on the particular model. Strings can be made semi-local, and thus unstable, by adding extra charged chiral multiplets [9], or by assuming that the Higgs fields also transform non-trivially under some non-abelian group [10]. If the gauge symmetry is already broken during inflation, there are no strings at all. This can be done by either adding a non-renormalisable term in the superpotential [11], extra GUT Higgs superfields [12], or a discrete symmetry [13].

In this paper we determine the parameter range for which standard F-term inflation is compatible with CMB data. The strings that form at the end of GUT F-inflation do not satisfy the Bogomolny bound. Therefore the string tension is a function of the Higgs and gauge field masses. This is in contrast with the F-term inflation models that emerge as an effective description of brane inflation [7], as the strings formed in these models do satisfy the Bogomolny bound. Note that supersymmetry is broken in the string core, and hence there are fermionic zero mode solutions bounded to the strings in the global SUSY case [14]. These zero modes disappear when gravity is included [15]. F-strings do not carry any current and thus the tension and energy-per-unit-length of the strings are equal [15, 16]. Our work improves in two ways on the existing literature [17 − 20]. First of all, for GUT strings we take the corrections to the string tension away from the Bogomolny limit into account. This enlarges the parameter space. Secondly, we include in a consistent manner all supergravity corrections to the potential. To do so we assume general hidden sector supersymmetry breaking. Possible dissipative corrections are also discussed.

The layout of this paper is as follows. In the next section we introduce the potential of standard F-term hybrid inflation, and include all SUSY breaking and SUGRA corrections. In section 3 we address the bounds on the symmetry breaking scale implied by the data.

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Apart from the "10%-bound" mentioned at the very beginning of this introduction, we also give the Kaiser-Stebbins bound, and the bound coming from pulsar observations. The density perturbations produced by hybrid inflation are discussed in section 4. Setting them equal to the observed spectrum gives the symmetry breaking scale M as function of the Higgs-inflaton coupling κ. This allows to translate the various bounds on M in bounds on κ. We derive an analytic expression for $M(\kappa)$ in the limit where one term dominates the potential. However, our approximation breaks down in the large coupling limit and numerical calculations are needed. Our numerical results are presented in section 5. We discuss both the constraints on GUT and on brane F-term inflation. Finally, in section 6 we discuss warm inflation, occurring when the the inflaton or Higgs fields can decay during inflation and the corresponding dissipative terms are important. Dissipation can ameliorate the CMB bounds.

2. Hybrid inflation — the potential

The superpotential for standard hybrid inflation is given by [21, 22]

$$W_{\rm inf} = \kappa S(\phi_+\phi_- - M^2)\,, \tag{2.1}$$

with S a gauge singlet superfield, and ϕ_+, ϕ_- Higgs superfields in complex conjugate representations of a gauge group G. In this paper, we use the same notation for the superfield and their scalar components. The coupling constant κ and the symmetry breaking scale M can be taken real and positive without loss of generality. The supersymmetric part of the scalar potential is given by

$$\begin{aligned} V_{\rm SUSY} &= \sum_b \left|\frac{\partial W}{\partial \phi_b}\right|^2 + \frac{g^2}{2}\sum_a\left(\sum_b \phi^\dagger_{b,i} t^{a,i}_j \phi^j_b\right)^2 \\ &= \kappa^2|\phi_+\phi_- - M^2|^2 + \kappa^2|S|^2(|\phi_+|^2+|\phi_-|^2) + V_D \end{aligned} \tag{2.2}$$

where the sum b is over all fields. t^a_i are the generators of G, $a = 1\ldots n$ where n is the dimension of G, and $i,j = 1\ldots\mathcal{N}$ where $\mathcal{N}$ is the dimension of the representation of the field ϕ_b. Vanishing of the D-terms enforces $|\phi_-| = |\phi_+|$. Assuming chaotic initial conditions the fields get trapped in the inflationary valley of local minima at $|S| > S_c = M$ and $\phi_- = \phi_+ = 0$, where G is unbroken. The potential is dominated by a constant term

$$V_0 = \kappa^2 M^4 \tag{2.3}$$

which drives inflation. Inflation ends when the inflaton drops below its critical value S_c (or when the second slow-roll parameter η equals unity, whatever happens first) and the fields roll toward the global SUSY minima of the potential $|\phi_+| = |\phi_-| = M$ and $S = 0$. During this phase transition the gauge group G is spontaneously broken down to a subgroup H. Cosmic strings form via the Kibble mechanism if the vacuum manifold G/H is simply connected [23]. If G is embedded in a GUT theory, or $G = \mathrm{U}(1)$ as is the case in effective D-brane models, cosmic strings form [4, 5].

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In the standard scenario the flatness of the tree level potential is lifted by loop corrections [22]. These do not vanish during inflation because $F_S \neq 0$ and SUSY is broken. The two scalar mass eigenstates $\chi_\pm = 1/\sqrt{2}(\phi_+ \pm \phi_-)$ have masses $m_\pm^2 = \kappa^2(S^2 \pm M^2)$, while their fermionic superpartners both have mass $\tilde{m}_\pm^2 = \kappa^2 S^2$. If the Higgs representation is $\mathcal{N}$-dimensional, there are $\mathcal{N}$ such mass-splitted double-pairs. The one loop correction to the potential can be calculated using the Coleman-Weinberg formula [24] $V_{\rm loop} = \frac{1}{64\pi^2}\sum_i (-)^{F_i}\, M_i^4 \ln\frac{M_i^2}{\Lambda^2}$, which for the superpotential in eq. (2.1) gives[1] [22]

$$V_{\rm loop} = \frac{\kappa^4 M^4 \mathcal{N}}{32\pi^2}\left[2\ln\left(\frac{M^2\kappa^2 z}{\Lambda^2}\right) + (z+1)^2\ln(1+z^{-1}) + (z-1)^2\ln(1-z^{-1})\right] \tag{2.4}$$

with

$$z = x^2 = \frac{|S|^2}{M^2}\,. \tag{2.5}$$

2.1 SUGRA corrections

In addition to $V_{\rm loop}$ there are SUGRA corrections to the potential, i.e., corrections that vanish in the limit that the Planck mass is taken to infinity and gravity decouples. In any model that aspires to describe the real world, there are two sources of SUSY breaking: SUSY breaking by the finite energy density during inflation and SUSY breaking in the true vacuum; the latter is responsible for the soft terms today. Both sources of breaking contribute to the SUGRA corrections. These corrections therefore depend on the particular scenario for SUSY breaking at low energy and in particular on the form of the hidden sector superpotential.

The superpotential gets a contribution from both the inflaton and hidden sector potential: $W_{\rm tot} = W_{\rm inf}(S,\phi_+,\phi_-) + W_{\rm hid}(z)$. In gauge mediated SUSY breaking models there is also a contribution from the messenger sector and in general GUT models from other GUT superfields. We assume that they do not couple to the inflaton sector except gravitationally. The hidden sector expectation values at the minimum of V may generically be written as

$$\langle z\rangle = a m_{\rm p}\,, \qquad \langle W_{\rm hid}\rangle = \mu m_{\rm p}^2\,, \qquad \left\langle \frac{\partial W_{\rm hid}}{\partial z}\right\rangle = c\mu m_{\rm p}\,, \tag{2.6}$$

with $m_{\rm p} = (8\pi G)^{-1/2} = 2.4\times 10^{18}\,$GeV the reduced Planck mass, a, c dimensionless numbers, and μ a mass parameter characterizing the VEV of the hidden-sector superpotential. Setting the cosmological constant to zero by hand ($\langle V\rangle = 0$ after inflation) requires tuning

$$|c + a^*|^2 = 3\,. \tag{2.7}$$

The scalar potential is

$$V = {\rm e}^{K/m_{\rm p}^2}\left[\sum_\alpha \left|\frac{\partial W}{\partial\phi_\alpha} + \frac{\phi_\alpha^* W}{m_{\rm p}^2}\right|^2 - 3\frac{|W|^2}{m_{\rm p}^2}\right] \tag{2.8}$$

[1]When $|S|$ is very close to $m_{\rm p}$ there are SUGRA corrections to the masses of the scalars and fermions which enter the loop correction. However, as we shall see further, the loop corrections dominate when $|S|$ is very close to $S_c \ll m_{\rm p}$, and these corrections do no play any rôle.

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where the sum is over all fields. We take minimal kinetic terms, corresponding to a Kähler potential $K = \sum_\alpha |\phi_\alpha|^2$. The true vacuum gravitino mass is then given by

$$m_{3/2} = \mathrm{e}^{|a|^2/2}\mu\,. \tag{2.9}$$

In gravity mediated SUSY breaking schemes $m_{3/2} \sim$ TeV is of the order of the soft mass terms, whereas it can be smaller in gauge mediated schemes.

During inflation when $|S| > S_c$ and the SUGRA F-term $F_\alpha = \partial W_{\rm tot}/\partial\phi_\alpha + \frac{\phi^*_\alpha W_{\rm tot}}{m_{\rm p}^2} \neq 0$ for $\phi_\alpha = z, S$. This is the SUGRA generalization of F-term SUSY breaking. Following ref. [25], we rescale the visible sector superpotential $W_{\rm inf} \to \mathrm{e}^{-|a|^2/2} W_{\rm inf}$ in order to recover from eq. (2.8) the properly normalized tree level potential in the global limit given by eq. (2.2), i.e., in the limit when $m_{\rm p} \to \infty$ and gravity decouples.

Expanding the exponential term in eq. (2.8) in powers of $|S|/m_{\rm p}$, we find the SUGRA corrections to $V_{\rm SUSY}$[2]

$$V_{\rm SUGRA} = V_{\rm A} + V_{\rm m} + V_{\rm NR}\,. \tag{2.10}$$

The A-terms are of the form

$$V_A = 2\kappa M^2 m_{3/2}|S|\cos(\arg\mu - \arg S)\left(2 + \frac{|S|^2}{m_{\rm p}^2} + \cdots\right) \tag{2.11}$$

with the ellipsis denoting higher powers of $|S|/m_{\rm p}$. The linear term is dominant. It is proportional to both the high and low energy SUSY breaking scale. This term was discussed in [17] in the context of an explicit O'Raifeartaigh model, and more recently in [18].

The mass term is of the form

$$V_m = \left(3H^2(|a|^2 + \cdots) - 2m_{3/2}^2\right)|S|^2 \tag{2.12}$$

where we have expanded the $\mathrm{e}^{|a|^2}$ in powers of $|a|$, which is only valid for $|a| \ll 1$; the ellipsis denote higher order terms in $|a|$. We will refer to the first term as the Hubble induced mass. The second term, the vacuum soft mass term, is negligible small compared to the other SUGRA contributions. The Hubble parameter during inflation is given by the Friedman equation: $H^2 = V/(3m_{\rm p}^2) \approx V_0/(3m_{\rm p}^2)$. There is no soft mass in the absence of low energy SUSY breaking ($a \to 0, m_{3/2} \to 0$). This is a consequence of taking minimal Kähler, the zeroth order term $V_m \sim H^2|S|^2$ cancels; this can be considered as fine tuning.

A scale invariant perturbation spectrum in agreement with observations is obtained for sufficiently small slow roll parameter $|\eta| \sim |m_S^2|/H^2 \lesssim 10^{-2}$ [26], see eq. (4.2) below. For generic Kähler potential $\eta \sim 1$, which is the infamous η-problem. Scale invariance thus requires minimal Kähler potential (or close to it) to cancel to the zeroth order Hubble induced mass term proportional to $|a|^0$, and $|a| \lesssim 0.1$ to cancel the higher order terms proportional to $|a|^{2n}$ as given in eq. (2.12). This last requirement excludes the simplest Polonyi model $W_{\rm hid} = M_s^2(\beta + z)$ with $z = \mathcal{O}(1)m_{\rm p}$.[3]

[2]We neglect higher order corrections to the Kähler and super potential; we expect these terms to change the coefficients in front of the various terms but not their qualitative structure. They could also destabilise the VEV of the Higgs fields and we assume here that they do not.

[3]The Hubble induced mass term in eq. (2.12) comes from an expansion of $\mathrm{e}^{K/m_{\rm p}^2}$ in eq. (2.8) to second order in the fields; this term is missed in [17].

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The non-renormalisable SUGRA terms are of the form

$$V_{NR} = \frac{1}{2}(3H^2 - m_{3/2}^2)|S|^2 \left(\frac{|S|^2}{m_{\rm p}^2} + \cdots \right) \tag{2.13}$$

with the ellipsis denoting higher order terms in $|S|^2/m_{\rm p}^2$. The Hubble induced term dominates. They have been discussed before [27].

How generic are the above SUGRA corrections? We assume minimal Kähler potential and general hidden sector SUSY breaking. A non-minimal Kähler potential would generically be catastrophic, as it means a large Hubble induced mass impeding slow roll inflation. The hidden sector is characterized by the scale $m_{3/2}$ and the dimensionless constant $|a|$. The A-terms are small for a small gravitino mass, as can occur in gauge mediated SUSY breaking where $m_{3/2} \ll$ TeV is possible. The Hubble induced mass term does not depend on the gravitino mass; its effect can only be decreased by taking $a \ll 1$. Note that this entails a large hierarchy between the dimensionless hidden sector parameters a and c, see eq. (2.7), implying some sort of fine tuning. The non-renormalisable term is independent of the SUSY breaking sector, and is generic. If SUSY breaking in the true vacuum occurs after inflation, i.e., z only acquires its VEV after inflation, then μ, $a \to 0$: the mass and A-terms are absent, but the NR terms are still there.

The scalar potential including all corrections then is

$$V = V_{\rm SUSY} + V_{\rm loop} + V_{\rm SUGRA} \,. \tag{2.14}$$

During inflation $V_{\rm SUSY} \approx V_0 = \kappa^2 M^4$, and the potential reads

$$\begin{aligned} V = \kappa^2 M^4 \Bigg[1 + \frac{\kappa^2 \mathcal{N}}{32\pi^2} \Bigg[2 \ln \left(\frac{\kappa^2 |S^2|}{\Lambda^2} \right) + (z+1)^2 \ln(1+z^{-1}) + \\ + (z-1)^2 \ln(1-z^{-1}) \Bigg] + \frac{1}{2} \frac{|S|^2}{m_p^2} \left(\frac{|S|^2}{m_{\rm p}^2} + \cdots \right) + \left(|a|^2 + \cdots \right) \frac{|S|^2}{m_p^2} \Bigg] + \\ + 2\kappa M^2 m_{3/2} |S| \cos(\arg\mu - \arg S) \left(2 + \frac{|S|^2}{m_{\rm p}^2} + \cdots \right), \end{aligned} \tag{2.15}$$

where we have omitted in $V_{\rm NR}$ and $V_{\rm m}$ the gravitino mass dependent terms which are negligible during inflation in the parameter range of interest. Keeping only the dominant terms and expressing everything in terms of the real inflaton field $\sigma = \sqrt{2}|S|$ gives

$$\begin{aligned} V = \kappa^2 M^4 \Bigg[1 + \frac{\kappa^2 \mathcal{N}}{32\pi^2} \Bigg[2 \ln \left(\frac{2\kappa^2 \sigma^2}{\Lambda^2} \right) + (z+1)^2 \ln(1+z^{-1}) + \\ + (z-1)^2 \ln(1-z^{-1}) \Bigg] + \frac{\sigma^4}{8 m_p^4} + \frac{|a|^2 \sigma^2}{2 m_p^2} \Bigg] + \\ + \kappa A m_{3/2} M^2 \sigma \end{aligned} \tag{2.16}$$

where $A = 2\sqrt{2}\cos(\arg\mu - \arg S)$. Here we have assumed that $\arg S$ is constant during inflation. Further, $z = x^2 = |S|^2/M^2 = \sigma^2/(2M^2)$ so that $z = x = 1$ when $\sigma = \sigma_c$.

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3. CMB constraints

3.1 String tension

Supersymmetry is broken inside the strings which form at the end of SUSY inflation, and hence there are fermionic zero mode solutions bounded to the strings in the global SUSY case [14]. These zero modes disappear when gravity is included [15]. F-strings do not carry any current and hence the tension and energy-per-unit length of the strings are equal [15, 16].

Cosmic strings satisfying the Bogomolny bound have a tension $\mu = 2\pi M^2$, with M the VEV of the string Higgs fields far away from the string. The strings forming at the end of F-term inflation do not satisfy this bound, and there are corrections to the simple formula above, which depend on the ratio of the common Higgs mass m_ϕ to the gauge boson mass m_A [28, 29]

$$\mu = 2\pi M^2 \epsilon(\beta) \tag{3.1}$$

where $\beta = (m_\phi/m_A)^2$. $m_\phi = \kappa M$ with κ the superpotential coupling constant and m_A is given in terms of gauge coupling constant g; the exact relation depends on the dimension and transformation properties of the Higgs fields ϕ_+ and ϕ_-. $m_A = \sqrt{2} g M$ for $\mathcal{N} = 1$. In GUT models $g^2 \simeq 4\pi/25$ and $m_A \simeq M$. In this paper we shall use $m_A \simeq 1$ unless stated otherwise. In the Bogomolny limit $\epsilon(1) = 1$. From ref. [28]

$$\epsilon(\beta) \approx \begin{cases} 1.04\beta^{0.195}\,, & \beta > 10^{-2}, \\ \dfrac{2.4}{\log(2/\beta)}\,, & \beta < 10^{-2}\,. \end{cases} \tag{3.2}$$

For $m_A \simeq M$, β varies from 1 to 10^{-12} as κ goes from 1 to 10^{-6}, and thus μ changes by a factor ~ 20. In figure 1 we plotted $\epsilon(\beta)$ as function of κ for $m_A = M$. For $\kappa \sim 10^{-2}$ the bound on μ, discussed below, is weaker by a factor 4 if $\epsilon(\beta)$ is taken into account.

3.2 Bounds from CMB

Cosmological perturbations from inflation and strings in hybrid models are proportional to the same scale M. They are uncorrelated and thus the multipole moments of the CMB power spectrum just add up [4]. The proportionality coefficients depend on the GUT parameters and on the normalised contribution of each component. Cosmic strings do not predict the acoustic peaks which have been measured in CMB experiments and hence their contribution should be rather small, less than 10% at the 3σ level [2]. The contribution from cosmic strings depends on the mass-per-unit-length μ and on the density of the string network at last scattering. No full field theoretic simulations exist and hence no robust prediction of the full power spectrum can be made right now. The model parameters can be constrained, but only up to the uncertainties of the simulations.

The cosmic string contribution to the quadrupole is given by

$$\left(\frac{\delta T}{T}\right)_{\rm cs} = y G \mu\,; \tag{3.3}$$

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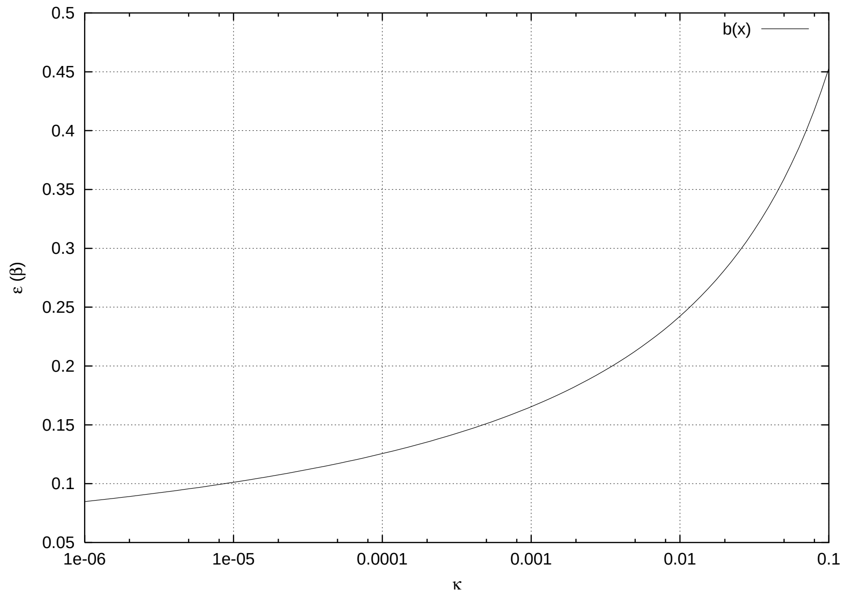

Figure 1: $\epsilon(\beta)$ vs. κ for $m_A \simeq M$.

the parameter y depends on the simulation. Recent work predicts $y = 8.9$ [30]. The error margin they quote gives a range $y = 6.7 - 11.6$. Older simulations give $y = 6$ [31], and semi-analytic approximations give $y = 3 - 6$ [32].

The quadrupole measured by COBE (which coincides with WMAP data) is [26, 33]

$$\left(\frac{\delta T}{T}\right)_{\rm COBE} = 6.6 \times 10^{-6}\,. \tag{3.4}$$

The cosmic string contribution to the quadrupole is given by

$$B = \left|\frac{(\delta T/T)_{\rm cs}}{(\delta T/T)_{\rm COBE}}\right|^2 . \tag{3.5}$$

The analysis of ref. [2] gives the bound $B < 0.1$, i.e., a string contribution less than 10%. Using eqs. (3.3) and (3.1) this implies

$$G\mu < 6.9 \times 10^{-7}\left(\frac{3}{y}\right) \quad \Rightarrow \quad M < 4.1 \times 10^{15}\sqrt{\frac{(3/y)}{\epsilon(\beta)}}\,. \tag{3.6}$$

In our numerical simulations we will use the conservative value $y = 3$. The bound on M is a factor $\sqrt{3}$ stronger for $y = 9$, the value suggested by the most recent simulations.

If there are extra dimensions the string reconnection probability p (the probability that two strings reconnect when they pass through each other), which is one for four dimensional gauge theory solitons, can be less than unity [6, 34]. The result is that more energy is stored in the string network at a given time, and thus the constraint on the string scale in eq. (3.6) becomes tighter by a factor $\sqrt{p}$ [2].

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Strings can also influence the CMB pattern after the time of last scattering, through the Kaiser-Stebbins effect [35]. If a moving string is traversing the photons coming towards us this will give rise to a step-like discontinuity on small angular scales, the step size being proportional to the mass-per-unit-length of the string. In ref. [36] Jeong & Smoot used the absence of such discontinuities to derive the bound

$$G\mu < 3.3 \times 10^{-7} \quad \Rightarrow \quad M < \frac{2.8 \times 10^{15}}{\sqrt{\epsilon(\beta)}} \,. \tag{3.7}$$

The Jeong-Smoot (JS)-bound is stronger than the 10%-bound for $y < 6.3$. The JS-bound depends less on the evolution of the string network, i.e., on the results of numerical simulations; in this sense it is a much stronger bound.

Finally, the string scale can be bounded by constraining the stochastic gravitational wave background produced by the string network [29]. Such a background would distort the regularity of pulsar timing. No such distortion has been observed [37], which translates to

$$G\mu < 1 \times 10^{-7} \quad \Rightarrow \quad M < \frac{1.5 \times 10^{15}}{\sqrt{\epsilon(\beta)}} \,. \tag{3.8}$$

Although the pulsar-bound is the most stringent of the three, it is also the one with the largest uncertainties. Due to the huge range of scales involved, one of the least certain aspects of cosmic string dynamics is the long term evolution of small scale structure. And it is this small scale structure that governs the gravitational radiation.

In our simulations, we shall use the three different bounds given in eqs. (3.6), (3.7) and (3.8). In our analytic formulas we will use $M < M_{\rm CMB} \approx 3 \times 10^{15}\,{\rm GeV}$, which corresponds to the JS-bound.

Before closing this section, we would like to mention that a lensing event consistent with a cosmic string has been reported recently [38]. Further observation is needed to determine whether the lensing is indeed string induced. Such a string would have

$$G\mu \geq 4 \times 10^{-7} \quad \Rightarrow \quad M \geq \frac{3 \times 10^{15}}{\sqrt{\epsilon(\beta)}} \,. \tag{3.9}$$

to produce the observed image separation. If it is indeed a cosmic string, we are on the verge of seeing it in the CMB data.

4. Density perturbations

We recall here the important formulae for the density perturbations [39]. The number of e-foldings before the end of inflation is

$$N_Q = \int_{\sigma_{\rm end}}^{\sigma_Q} \frac{1}{m_{\rm p}^2} \frac{V}{V'} {\rm d}\sigma \tag{4.1}$$

where the prime denotes derivation with respect to the normalised real scalar field $\sigma \equiv \sqrt{2}|S|$, and the subscript Q denotes the time observable scales leave the horizon, which happens $N_Q \approx 60$ e-folds before the end of inflation. $\sigma_{\rm end}$ is the inflaton VEV when

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inflation ends, which is either the critical point where the Higgs mass becomes tachyonic $\sigma_{\rm end} = \sigma_c = \sqrt{2}M$, or the value for which one of the slow roll parameters

$$\epsilon = \frac{1}{2}m_{\rm p}^2\left(\frac{V'}{V}\right)^2, \qquad \eta = m_{\rm p}^2\left(\frac{V''}{V}\right), \tag{4.2}$$

exceeds unity. For hybrid inflation $\epsilon \ll \eta$ and can be neglected. The second slow roll parameter η blows up in the limit $\sigma \to \sigma_c$ (as a consequence of the field $\chi_- = (\phi_+ - \phi_-)/\sqrt{2}$ becoming massless), and thus determines the end of inflation. As we will see, for small enough coupling $\sigma_{\rm end} \approx \sigma_c$. The inflaton contribution to the CMB quadrupole anisotropy is

$$\left(\frac{\delta T}{T}\right)_{\rm inf} = \frac{1}{12\sqrt{5}\pi m_{\rm p}^3}\frac{V^{3/2}}{V'}, \tag{4.3}$$

evaluated at $\sigma = \sigma_Q$. The tensor contribution $(\delta T/T)_{\rm tens} \approx 0.03 H_*/m_{\rm p}$ is small, and can be neglected. The total anisotropy is

$$\left(\frac{\delta T}{T}\right) = \sqrt{\left(\frac{\delta T}{T}\right)_{\rm inf}^2 + \left(\frac{\delta T}{T}\right)_{\rm cs}^2 + \left(\frac{\delta T}{T}\right)_{\rm s}}. \tag{4.4}$$

Here we have included a possible contribution from a scalar field different from the inflaton, denoted by $(\delta T/T)_{\rm s}$. This term can be important if alternative mechanisms for density perturbation are at work, as is the case in the curvaton [40] and inhomogeneous reheat scenario [41]. In these scenarios the fluctuations of the curvaton field respectively the field modulating the inflaton decay rate give the dominant contribution to the density perturbations. We define

$$\delta_C = \frac{(\delta T/T)_{\rm inf} + (\delta T/T)_{\rm cs}}{(\delta T/T)_{\rm COBE}}. \tag{4.5}$$

If the inflaton sector including cosmic strings is the only source of perturbations $\delta_C = 1$, whereas $\delta_C \ll 1$ in the curvaton and inhomogeneous reheat scenario. Note that the curvaton scenario in its simplest form can only work for $H \sim \sqrt{V_0/m_{\rm p}^2} \gtrsim 10^7$ GeV [42, 43].

Finally, the spectral index is given by

$$n_s - 1 = -6\epsilon + 2\eta \tag{4.6}$$

WMAP bounds $n_s = 0.99 \pm 0.04$ [26].

The potential energy V_0 dominates and drives inflation. The amplitude of the density perturbations is set by V'. The spectral index is determined by the second derivative V''. The strategy to compute the perturbation spectrum is the following. First determine the inflaton VEV when inflation ends from the condition $|\eta(\sigma_{\rm end})| = 1$. Then use eq. (4.1) to find the inflaton VEV when observable scales leave the horizon σ_Q. Finally, compute the quadrupole using eqs. (4.3), (4.4) evaluated at $\sigma = \sigma_Q$. Setting the quadrupole equal to the observed COBE value gives the symmetry breaking scale M as a function of the superpotential coupling κ.

In various regions of the parameter space, various corrections dominate. An analytic approximation to the perturbation spectrum is then possible in the small coupling limit (where $x_{\rm end} \approx x_Q \approx 1$), as we will discuss now.

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4.1 The loop regime

Assume first that the loop potential $V_{\rm loop}$ dominates. The derivative of the potential, which is needed for both the determination of N_Q and $\delta T/T$, is [22]

$$V'_{\rm loop} = \frac{\kappa^4 M^3 \mathcal{N}}{8\sqrt{2}\pi^2} x f(x^2)\,, \tag{4.7}$$

with $x = \sigma/\sigma_c$ and

$$f(z) = (z+1)\ln(1+z^{-1}) + (z-1)\ln(1-z^{-1})\,. \tag{4.8}$$

Recall that in hybrid inflation the first slow roll parameter is negligible with respect to the second $\epsilon \ll \eta$. To determine the end of inflation we calculate the x-value for which $|\eta|$ becomes unity. The loop contribution to η is

$$\eta_{\rm loop} = \frac{\kappa^2 \mathcal{N}}{16\pi^2}\left(\frac{m_{\rm p}}{M}\right)^2 g(x^2) \tag{4.9}$$

with

$$g(z) = (3z+1)\ln(1+z^{-1}) + (3z-1)\ln(1-z^{-1})\,. \tag{4.10}$$

If $x \gg 1$, $g(x^2) \simeq -3x^{-2} + \mathcal{O}(x^{-4})$ which implies that $x_{\rm end} = \sqrt{3\mathcal{N}/(16\pi^2)}\kappa(m_{\rm p}/M)$. This is the regime where the inflaton VEV during inflation is large, and consequently the non-renormalisable potential $V_{\rm NR}$ becomes important. For this reason, we will not discuss it any further.

The above expression breaks down for small enough coupling $\kappa \lesssim 7/\sqrt{\mathcal{N}}(M/m_{\rm p}) \lesssim 10^{-2}/\sqrt{\mathcal{N}}$, (in the last step we used $M \sim 10^{16}\,\mathrm{GeV}$, from eq. (4.13) below) and $x_{\rm end}$ approaches unity. Using $x_{\rm end} = 1$ in eq. (4.1) we find

$$N_Q = \frac{16\pi^2}{k^2\mathcal{N}}\left(\frac{M}{m_{\rm p}}\right)^2 \int_1^{x_Q} \frac{1}{x f(x)}\mathrm{d}x\,. \tag{4.11}$$

In the limit where the factor in front of the integral $16\pi^2/(k^2\mathcal{N})(M/m_{\rm p})^2 \gg N_Q$, the integral has to be much less than unity which requires $x_Q \to x_{\rm end} \approx 1$. We conclude that we can approximate

$$x_Q \approx x_{\rm end} \approx 1\,, \qquad \text{for } \kappa \lesssim \sqrt{\frac{16\pi^2}{N_Q\mathcal{N}}}\left(\frac{M}{m_{\rm p}}\right) = \frac{10^{-2} - 10^{-3}}{\sqrt{\mathcal{N}}}\,. \tag{4.12}$$

The inflaton VEV during inflation is always small, and NR terms can be negligible. Using $x_Q \approx 1$ in the expression for the fluctuations eq. (4.3), and setting it equal to the observed value, allows to extract M

$$M_{\rm loop} = 2\times 10^{-2}(\delta_C \mathcal{N}\kappa)^{1/3} m_{\rm p}\,, \tag{4.13}$$

where the subscript *loop* is a reminder that this formula is valid in the domain where the loop potential is the dominant term in V'. The parameter δ_C defined in eq. (4.5) gives the normalised contribution of the inflaton sector including cosmic strings to the CMB. If the

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inflaton sector including cosmic strings is the only source of perturbations $\delta_C = 1$, whereas $\delta_C \ll 1$ in the curvaton and inhomogeneous reheat scenario. The tensor perturbations are negligible, and in the parameter space of interest, also the string contribution is small, less than 10%.

In the loop dominated regime M is a single valued function of κ, given by eq. (4.13). This expression is valid in the small coupling regime $\kappa < 10^{-2} - 10^{-3}/\sqrt{N}$, where it is a good approximation to ignore the string contribution to the CMB, and where $x_Q \approx 1$ is valid.

The CMB constraint $M_{\rm loop} < M_{\rm CMB} \approx 3 \times 10^{15}/\sqrt{\epsilon(\beta)}$ discussed in section 3 implies

$$\kappa < 2 \times 10^{-4} \frac{1}{\delta_C \mathcal{N} \epsilon(\beta)^{3/2}} . \tag{4.14}$$

Taking into account $\epsilon(\beta)$ — the correction to the mass-per-unit-length away from the Bogomolny bound — weakens the bounds on the coupling compared to an analysis in which this correction is ignored, as was done in ref. [19]. This correction changes the bound by a factor $\epsilon(\beta)^{-3/2} \sim 10$ to $\kappa \lesssim 10^{-3}$, where we have used that $\epsilon \sim 1/5$ for $\kappa \sim 10^{-3}$. Note, however, that the analytic approximation breaks down in this limit, and one has to go to numerical calculations for a more precise bound.

$V_{\rm loop}$ is always present. At small coupling it decreases rapidly ($\propto \kappa^4$) and $V_A, V_m, V_{\rm NR}$ can become dominant. In the large coupling regime $S \to m_{\rm p}$ and $V_{\rm NR}$ dominates. Note that in the regimes where other contributions dominate, the mass scale M is higher than it would be in the presence of just the loop potential, and the CMB constraints are stronger. The reason is that $(\delta T/T) \propto M^6/V'$, and thus the larger V' the larger M is needed to obtain the observed temperature anisotropy.

4.2 The non-renormalisable SUGRA regime

The non-renormalisable corrections dominate over the loop corrections for density perturbations if $V'_{\rm NR} > V'_{\rm loop}$, when

$$\frac{x^2}{f(x^2)} > \frac{\kappa^2 \mathcal{N}}{16\pi^2} \left(\frac{m_{\rm p}}{M}\right)^4 . \tag{4.15}$$

There are two regimes where this inequality is satisfied. The first is when the inflaton VEV, and thus x, is large during inflation. For $x \gg 1$ the term $x^2/f(x^2) \to x^4$ and the l.h.s. of the above equation is large. For very small κ the r.h.s. becomes small, and this is the other regime where the NR potential can become important.

First consider the large coupling regime where $x \gg 1$. The contribution of V_{NR} to η is $\eta_{NR} = 3(M/m_{\rm p})^2 x^2$. The slow roll parameter exceeds unity for large x and drops below one when $x = 1/\sqrt{3}(m_{\rm p}/M)$. Thus inflation can only happen for $\sigma = \sqrt{2} M x < (2/3)^{1/2} m_p$. At still lower x, the loop potential starts dominating η; inflation ends when $\eta_{\rm loop}$ given by eq. (4.9) becomes unity. Hence

$$0.2 m_{\rm p} \sqrt{\mathcal{N}} \kappa = \sigma_{\rm end} < \sigma_Q < 8 \times 10^{-2} m_{\rm p} \tag{4.16}$$

where for the upper bound we used that $|\eta| < 10^{-2}$ when observable scales leave the horizon, to assure scale invariance in accordance with the WMAP data [26]. Both inequalities

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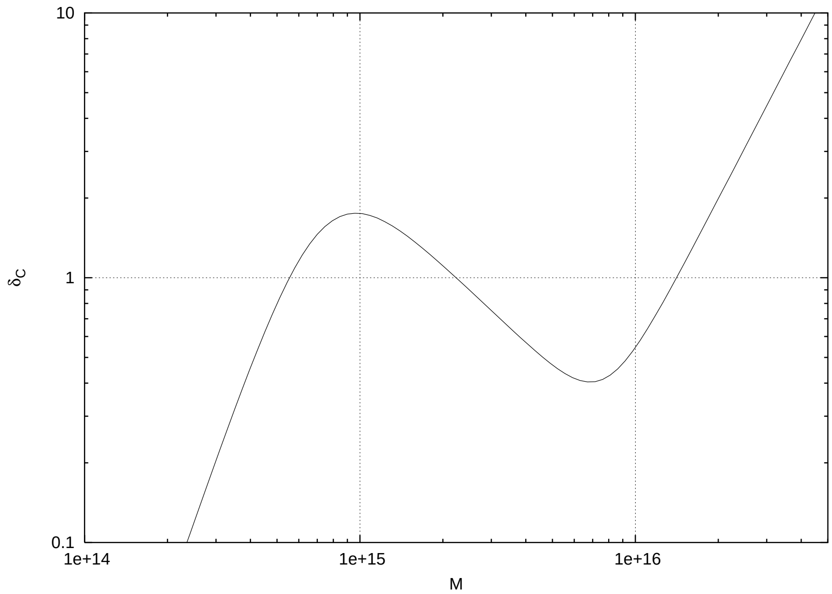

Figure 2: δ_C vs. M for $\kappa = 10^{-6}$ and $\mathcal{N} = 1$.

together give the upper upper bound $\kappa < 0.5/\mathcal{N}$ obtained in the limit $x_Q \to x_{\rm end}$. One should realize however, that 60 e-folds should pass when the inflaton rolls from x_Q to $x_{\rm end}$, and they cannot be taken arbitrarily close together.

An analytical approximation in the large coupling regime is hard because of a non-trivial expression for σ_Q, and because the string contribution to the CMB becomes important. However, in the range where non-renormalisable corrections dominate, the scale M is higher than $M_{\rm loop}$ given in eq. (4.13), which is excluded by CMB measurements, i.e., by the requirement $M < M_{\rm CMB}$. The spectral index is

$$n_s - 1 \simeq 2\eta = -\frac{3\sigma_Q^2}{m_{\rm p}^2} + \frac{3\kappa^2 \mathcal{N} m_{\rm p}^2}{4\pi^2 \sigma_Q^2}\,; \tag{4.17}$$

it goes from negative to positive as the non-renormalisable contribution comes to dominate. The running of the spectral index is small [44]: $\mathrm{d}n_s/\mathrm{d}\ln\kappa \sim -10^{-3}$.

In the small coupling regime where $\kappa < 3/\sqrt{\mathcal{N}}(M/m_{\rm p})^2$ the NR potential dominates over the loop potential. As before, for small κ it is a good approximation to take $x_Q \approx x_{\rm end} \approx 1$ (see eq. (4.12)). Then eq. (4.3) gives

$$M_{\rm NR} = 3 \times 10^{14}\,\mathrm{GeV}\left(\frac{\kappa}{10^{-7}}\right)\delta_C^{-1}\,. \tag{4.18}$$

It is possible to have for one value of κ several solutions for M, which all produce the correct density perturbations. The reason is that for fixed κ, by varying M, different contributions dominate the density perturbations. At small M the loop potential dominates and $(\delta T/T) \propto M^3$, at larger M the NR contribution becomes important and $(\delta T/T) \propto M^{-1}$, and at still larger M the main contribution comes from cosmic strings and $(\delta T/T) \propto M^2$. An example is given in figure 2, where we have plotted the δ_C — the temperature

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fluctuation normalized by the COBE value as defined in eq. (4.5) — as a function of M for $\kappa = 10^{-6}$. The maximum M_1 corresponds to the mass scale where $V'_{\rm loop} = V'_{\rm NR}$, whereas the minimum M_2 corresponds to the scale where $(\delta T/T)_{\rm inf} = (\delta T/T)_{\rm cs}$. Now

$$
\begin{aligned}
M_1 &= 0.3\kappa^{1/2}\mathcal{N}^{1/4}m_{\rm p} \\
M_2 &= 0.3\left(\frac{0.9}{y\epsilon}\right)^{1/3}\kappa^{1/3}m_{\rm p}
\end{aligned}
\tag{4.19}
$$

where in the second line we have used that $\epsilon \sim 10^{-1}$ for small κ. For simplicity we have set $\delta_C = 1$, and assume that no alternative mechanism for density perturbations such as the curvaton scenario is at work. We will return to this point at the end of the subsection. We define κ_1 the coupling for which $\delta_C(M_1) = 1$, κ_2 the coupling for which $\delta_C(M_2) = 1$.

$$
\begin{aligned}
\kappa_1 &= 6\times 10^{-8}\mathcal{N}^{1/2} \\
\kappa_2 &= 4\times 10^{-6}\left(\frac{0.9}{y\epsilon}\right)^{1/2} .
\end{aligned}
\tag{4.20}
$$

There are then 3 possibilities.

1. $\delta_C(M_1) > 1$ and $\delta_C(M_2) > 1 \quad \Longleftrightarrow \quad \kappa > \kappa_1,\ \kappa_2$

   There is only one solution $M(\kappa)$ for which the loop potential dominates. It is given by $M_{\rm loop}$ in eq. (4.13). Note that at still larger coupling the contribution from NR terms and strings kick in again.

2. $\delta_C(M_1) > 1$ and $\delta_C(M_2) < 1 \quad \Longleftrightarrow \quad \kappa_2 > \kappa > \kappa_1$

   There are three solutions. One at low scale where the loop potential dominates and M is given by $M_{\rm loop}$ of eq. (4.13), a middle one where NR terms dominate and M is given by $M_{\rm NR}$ of eq. (4.18), and one at high scale which is dominated by string contributions. The latter solution is excluded by CMB data.

3. $\delta_C(M_1) < 1$ and $\delta_C(M_2) < 1 \quad \Longleftrightarrow \quad \kappa < \kappa_1,\ \kappa_2$

   There is one solution, dominated by the string contribution. This solution is excluded by CMB data.

Note that $\kappa > \kappa_1, \kappa_2$ is a lower bound on the coupling to be consistent with CMB data. Thus in the presence of NR terms, the coupling cannot be arbitrarily small. The minimum mass scale is $M_{\rm min} = M_{\rm loop}(\kappa_1) = 2\times 10^{14}\,{\rm GeV}\sqrt{\mathcal{N}}$.

The above discussion is illustrated by the numerical results shown in figure 3. Matching the quadrupole to the observed value gives M as a function of κ. The result with $V_{\rm loop}$, $V_{\rm NR}$ and the string contribution all turned on is given by the solid line. Also plotted are the result with $V_{\rm NR}$ turned off (dashed line) and with the string contribution turned off (dotted line). Consider the full solution, the solid line. At large coupling both the NR and string contribution are important. There is no solution for arbitrarily large κ, in agreement with the discussion below eq. (4.16). Going to smaller coupling, the loop potential starts to dominate and $x_Q \to 1$: the solution is a straight line as given by $M_{\rm loop}$ in eq. (4.13). The

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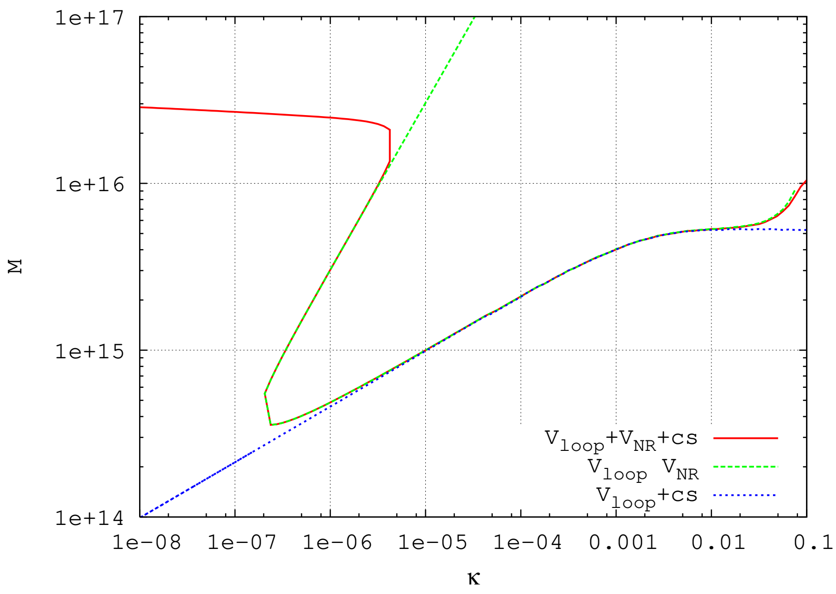

Figure 3: M vs. κ for $\mathcal{N} = 1$.

solution $M_{\rm loop}$ extends at low coupling until $\kappa = \kappa_1 \sim 5 \times 10^{-7}$. The second branch, where NR terms dominate, is between $5 \times 10^{-7} \sim \kappa_1 < \kappa < \kappa_2 \sim 5 \times 10^{-6}$. The upper branch is string dominated and excluded by CMB data. In the presence of NR terms agreement with the CMB requires $\kappa > \kappa_1$ and $M_{\rm min} \sim 2 \times 10^{14}\,{\rm GeV} < M_{CMB}$.

Let us now briefly return to the possibility that alternative mechanisms for density perturbations are at work and $\delta_C \ll 1$. As can be seen from figure 2, for small δ_C there is only one solution which is dominated by the loop contribution, and thus given by $M_{\rm loop}$ in eq. (4.13).

4.3 The Hubble regime

Consider the Hubble induced mass term in $V_{\rm m}$. This term dominates the density perturbations for $V'_{\rm m} > V'_{\rm loop}$, or

$$\kappa < \frac{4\pi|a|}{\sqrt{\mathcal{N} f(x^2)}} \left(\frac{M}{m_{\rm p}}\right) \approx 10^{-1} \frac{|a|^{3/2}\delta_C^{1/2}}{\mathcal{N}^{1/4}} \,. \qquad (4.21)$$

In the second equality we took $M = M_{\rm loop}$ and $f(x^2) \sim 1.4$, which is valid for $\kappa < 10^{-2} - 10^{-3}/\sqrt{N}$. The Hubble induced term dominates for small enough κ. If domination happens for $\kappa < \kappa_1 \sim 5 \times 10^{-6}$, the region already excluded by CMB date due to NR terms, the Hubble induced mass plays no rôle. This is for $|a| < a_0 \sim 10^{-3}(\mathcal{N}/\delta_C^2)^{1/6}$.

Consider then $|a| > a_0$; the Hubble induced mass becomes important before NR terms kick in. The contribution of the Hubble induced mass to η is constant $\eta_m = |a|^2$; scale invariance requires $|a| < 0.1$. The end of inflation is determined by the loop contribution, and $x_{\rm end} \approx 1$. Further $x_Q \approx 1$ for small coupling. Plugging this into eq. (4.3) gives

$$M_{\rm m} = 8 \times 10^{-4} m_{\rm p} \frac{|a|^2 \delta_C}{\kappa} \,. \qquad (4.22)$$

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In the κ-region where $V'_{\rm m}$ dominates M is a single valued function of κ. Since $M_{\rm m}$ grows with decreasing coupling, whereas $M_{\rm loop}$ decreases with decreasing coupling, there is a minimum scale M, obtained for $V'_{\rm loop} = V'_{\rm m}$, or equivalently, for $M_{\rm loop} = M_{\rm m}$:

$$M_{\rm min} = 2 \times 10^{16} \sqrt{|a|\delta_C} \mathcal{N}^{1/4} \,. \tag{4.23}$$

Agreement with CMB data requires $M_{\rm min} > M_{\rm CMB} \sim 3 \times 10^{15}\,{\rm GeV}/\sqrt{\epsilon(\beta)}$, or

$$|a| < \frac{2 \times 10^{-2}}{\sqrt{\mathcal{N}}\delta_C} \frac{1}{\epsilon(\beta)} \,. \tag{4.24}$$

This gives a stronger bound on $|a|$ than the requirement of scale invariance.

4.4 The A-regime

The A-term breaks the discrete symmetry $S \leftrightarrow -S$. For $A < 0$ there is a minimum for the potential at $S_0 \neq 0$. If $S_0 > S_c$ the inflaton gets trapped in the false vacuum leading to eternal inflation. Assume $V = V_0 + V_{\rm loop} + V_{\rm A}$, and all other terms negligible small. Then this happens for $x_0 > x_c$ with $x_0 \sim (\mathcal{N}\kappa^3 M)/(8\pi^2 A m_{3/2})$ and $x_c = 1$, which gives

$$M \gtrsim \frac{8\pi^2 A}{\mathcal{N}} \frac{m_{3/2}}{\kappa^3} \sim \frac{10^{16}\,{\rm GeV}}{\mathcal{N}} \left(\frac{m_{3/2}}{10^2\,{\rm GeV}}\right) \left(\frac{10^{-4}}{\kappa}\right)^3 . \tag{4.25}$$

Hence for a negative A-term, this is a problem for large κ/small gravitino mass.

No new minimum arises for positive A-term, which is the case we discuss now. These results equally apply to a negative A-term provided $x_0 \ll 1$. The A-term dominates the density perturbations if $V'_A > V'_l$, or

$$\kappa < 4 \left(\frac{m_{3/2}A}{\mathcal{N}M}\right)^{1/3} \sim \frac{4 \times 10^{-5}}{\mathcal{N}^{2/5}\delta_C^{1/10}} \left(\frac{m_{3/2}A}{\rm GeV}\right)^{3/10} \tag{4.26}$$

where in the second step we used $M \sim M_{\rm loop}$. A-term domination at small coupling happens before NR terms become important, unless the gravitino mass is small $m_{3/2} < 5 \times 10^{-4}\,{\rm GeV}(\mathcal{N}/A)$. Consider then $m_{3/2}$ large enough for the linear term to play a rôle. The A-term does not contribute to η, and $x_{\rm end} \approx 1$ determined by the loop contribution. In addition $x_Q \approx 1$ for small coupling. Using eq. (4.3), the observed perturbations are obtained for

$$M_A = \frac{4 \times 10^{13} \delta_C^{1/4}}{\sqrt{k}} \left(\frac{m_{3/2}A}{10^3\,{\rm GeV}}\right)^{1/4} . \tag{4.27}$$

When the linear term dominates, M increases as a function of decreasing coupling. This gives a lower bound on the mass scale M, obtained for $V'_A = V'_l$, which is

$$M_{\rm min} = 2 \times 10^{15}\,{\rm GeV} \mathcal{N}^{1/5} \delta_C^{1/20} \left(\frac{m_{3/2}A}{10^3\,{\rm GeV}}\right)^{1/10} . \tag{4.28}$$

This is only consistent with the CMB bound, i.e., with the requirement $M_{\rm min} > M_{\rm CMB}$, for $\mathcal{N} = 1$ and $m_{3/2} \lesssim 10^3\,{\rm GeV}$. Note that the dependence on the gravitino mass $m_{3/2}$ is weak, and very small $m_{3/2}$ is needed to lower this minimum scale considerably. This favors gauge mediated SUSY breaking, or a scenario in which hidden sector SUSY breaking happens after inflation.

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5. Numerical results

We have also solved the equations pertaining the density perturbations eqs. (3.3), (4.1), (4.2), (4.3), (4.4) numerically. For $\epsilon(\beta)$ we use the second expression in eq. (3.2). This introduces a factor 4 error in the limit $\beta = (\kappa/g)^2 \to 1$, where the BPS limit $\epsilon(1) = 1$ should be approached. Further we take $y = 3$, where y is the parameter which parameterises the string contribution as given in eq. (3.3), and reconnection probability $p = 1$. Setting the quadrupole eq. (4.4) equal to the observed value given in eq. (3.4) gives the symmetry breaking scale M as a function of κ. All other quantities such as the spectral index and the inflaton VEV when observable scales leave the horizon can also be computed. In this section we discuss the results.

Figure 4 shows M vs. κ for $\mathcal{N} = 1, 16, 126$. The potential includes the loop potential and the NR terms; all other terms are turned off. Also shown are the bounds on the scale M; from top to bottom these are the 10%-bound, the JS-bound, and the pulsar bound. The 10% bound and especially the pulsar bound has large theoretical uncertainties. At large and small coupling cosmic strings dominate the density perturbations, or equivalently $M(\kappa)$ exceeds the various bounds. This is the region excluded by experiments. At intermediate coupling, and for small enough $\mathcal{N}$, there are then two branches compatible with CMB data, the branch where the loop potential dominates, and a second branch at small coupling where the NR terms dominate. This in good agreement with the discussion in section 4.2. The κ range compatible with all bounds is $10^{-6} \lesssim \kappa \lesssim 10^{-3}/\mathcal{N}$. The upper bound lies in the κ-range where $V'_{\rm loop}$ dominates, which is why it has large $\mathcal{N}$ dependence. If we drop the pulsar bound, the parameter range is extended at large coupling to $10^{-7}/\mathcal{N} \lesssim \kappa \lesssim 10^{-2}/\mathcal{N}$. Only a small window remains for $\mathcal{N} = 126$, which is clearly disfavored.

In figure 5 we show the spectral index for the parameters of figure 4. The spectral index is less than one, except at large coupling when NR terms start to dominate the second derivative, see eq. (4.17). The coupling for which the spectral index starts to diverge corresponds to the upper bound on κ for which a solution exists (compare figures 4 and 5). This is in agreement with the discussion below eq. (4.16), where it should be noted that the potential is steep for large η and thus $x_{\rm end}$ and x_Q are well separated. The spectrum is indistinguishable from scale invariance and gives no new constraints on the available parameter space.

The string contribution to the quadrupole is parametrized by the parameter y. The value for y found in the literature ranges from $y = 3 - 12$. Figure 6 shows how this affects the 10%-bound. Plotted is the string contribution to the quadrupole B defined in eq. (3.5) as a function of κ for $\mathcal{N} = 1$ and different values of y. The 10% bound corresponds to $B = 0.1$. For $y = 3$ the bound on κ corresponds to that found from the $M(\kappa)$ plot in figure 4, as it should. For $y = 9, 12$ the 10%-bound is stronger, and the upper bound on κ is decreased by about a factor 10. We want to stress that in contrast to the 10% bound, the JS and pulsar bound are rather insensitive to y. And thus the κ-range compatible with the JS and pulsar bound is practically the same for the different y-values.

Figure 7 shows the results when the Hubble induced mass term is included, the different lines correspond to different values of $|a| = 10^{-1}, 10^{-2}, 10^{-3}$; in all cases $\mathcal{N} = 1$. $M(k)$

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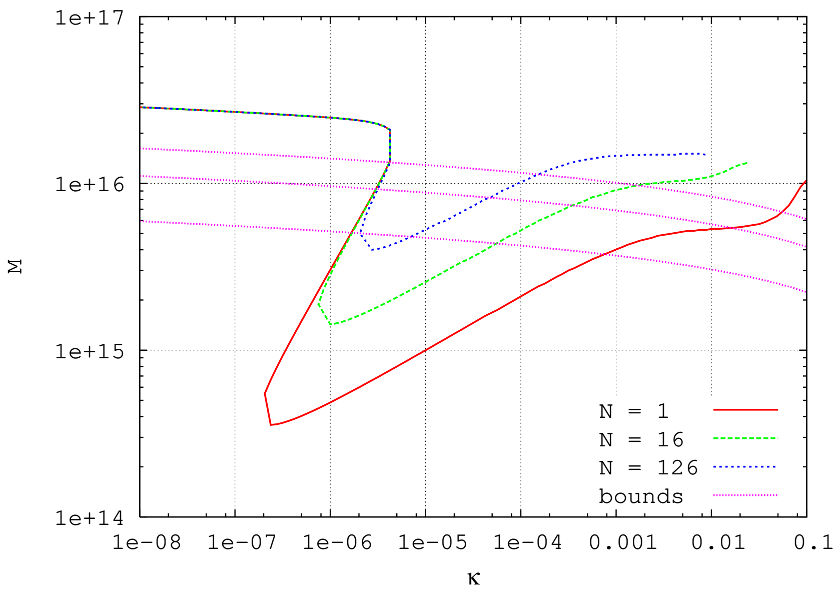

Figure 4: M vs. κ for $\mathcal{N} = 1, 16, 126$. Further shown are, from top to bottom, the 10%-bound, the JS-bound and the pulsar bound.

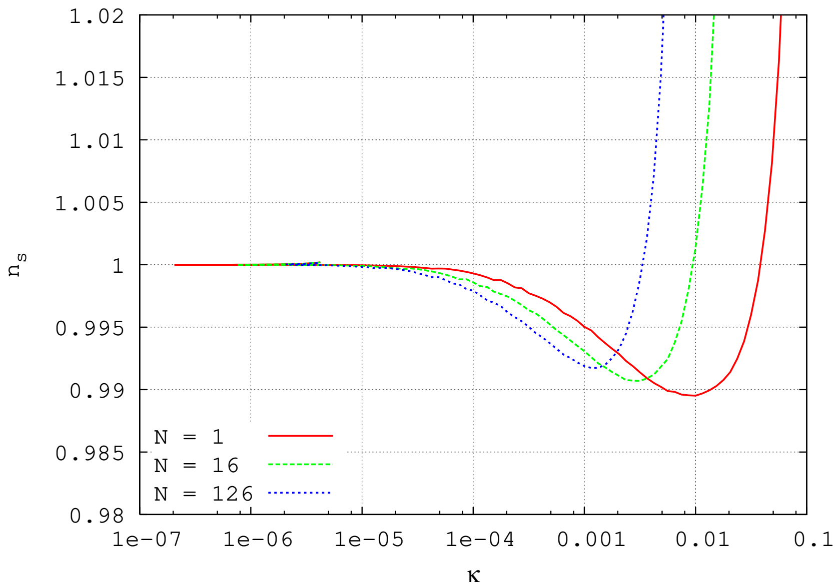

Figure 5: Spectral index n vs. κ for $\mathcal{N} = 1, 16, 126$.

increases with increasing $|a|$. Indeed too large $|a| = 10^{-1}$ is excluded by the data, whereas $|a| \sim 10^{-2}$ decreases the upper bound considerably, to $\kappa \lesssim 10^{-5}$. For $|a| \lesssim 10^{-3}$, the available parameter space is unaltered. V'_m is independent of $\mathcal{N}$, and we can get a good handle on how it affects parameter space for different $\mathcal{N}$ by comparing figures 4 and 7. For $\mathcal{N} = 16, 126$ it follows that $|a| \gtrsim 10^{-2}$ is excluded, whereas for example $|a| \sim 10^{-3}$ reduces the upper bound to $\kappa \lesssim 10^{-5}, 10^{-6}$.

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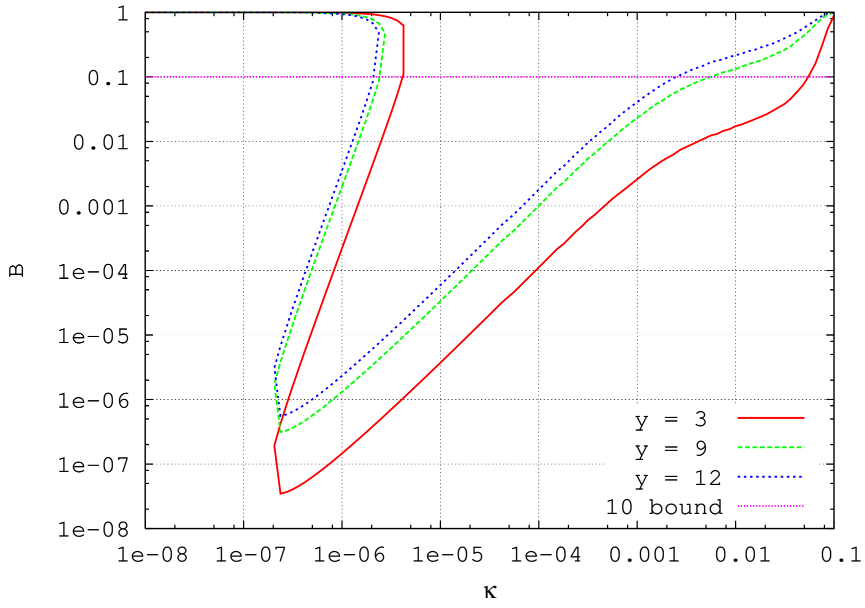

Figure 6: B vs. κ for $\mathcal{N} = 1$ and $y = 3, 9, 12$. $B = 0.1$ corresponds to the 10%-bound.

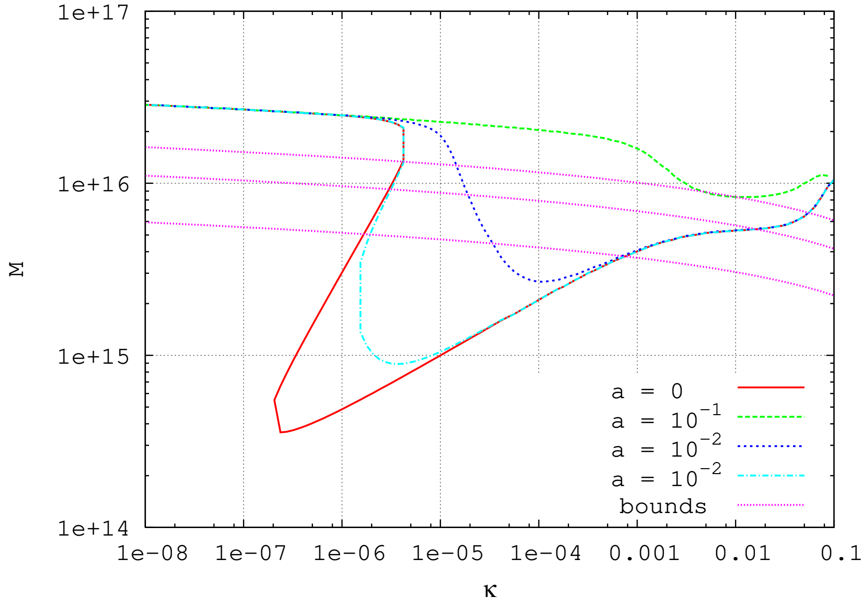

Figure 7: M vs. κ with Hubble mass term included. The plots are for $a = 10^{-1}, 10^{-2}, 10^{-3}$. Also shown are, from top to bottom, the 10% bound, the JS-bound and the pulsar bound.

Figure 8 shows the effect of including the linear A-term for gravitino masses $m_{3/2} = 10^3, 10^2, 10^0, 10^{-2}\,\mathrm{GeV}$ and $\mathcal{N} = 1$. A gravitino mass $m_{3/2} \gtrsim 10^2\,\mathrm{GeV}$ as obtained in gravity mediated SUSY breaking shrinks parameter space considerably: for $\mathcal{N} = 1$ the allowed range of κ is $10^{-5} - 10^{-4} \lesssim \kappa \lesssim 10^{-3} - 10^{-2}$, whereas no parameter space is left for $\mathcal{N} = 16, 126$.

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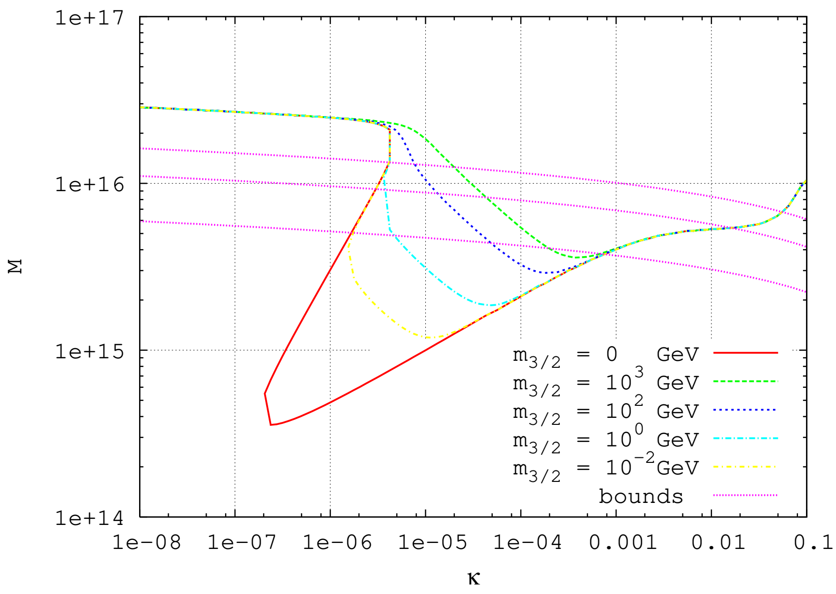

Figure 8: M vs. κ with A term included. The plots are $m_s = 10^3, 10^2, 10^0, 10^{-1}$. Also shown are, from top to bottom, the 10% bound, the JS-bound and the pulsar bound.

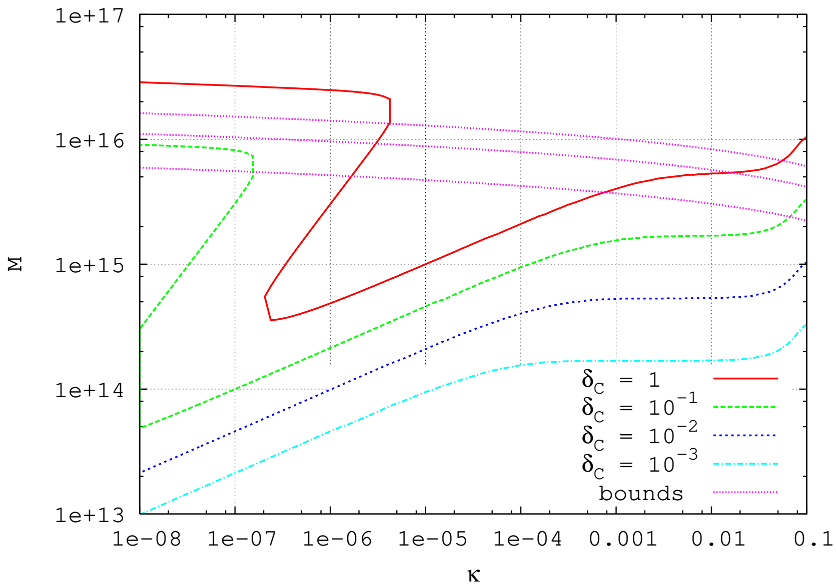

Figure 9: M vs. κ for $\delta_C = 10^{-1}, 10^{-2}, 10^{-3}$. Also shown are, from top to bottom, the 10% bound, the JS-bound and the pulsar bound.

Finally we consider the possibility that another scalar field than the inflaton gives the main contribution to the density perturbations, as occurs in the curvaton or inhomogeneous reheat scenario. If figure 9 we plot κ vs. M for $\delta_C = 10^{-1}, 10^{-2}, 10^{-3}$. Already when the combined contribution of the inflaton and cosmic strings to the quadrupole is reduced to 10% of the COBE value (and thus the other 90% must be provided by some other

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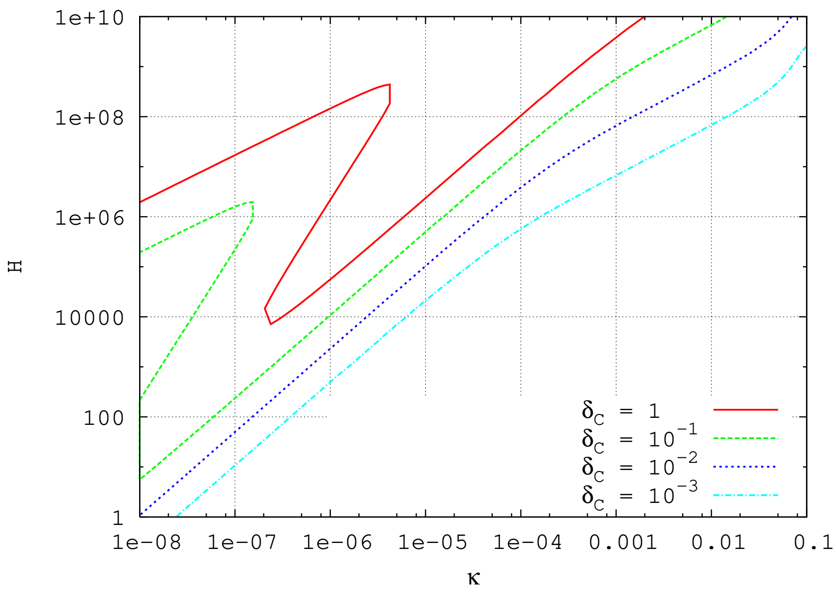

Figure 10: H vs. κ for $\delta_C = 10^{-1}, 10^{-2}, 10^{-3}$.

scalar field), are all constraints avoided. Invoking an alternative scalar field to explain the density perturbations is thus a good way to avoid all constraints on the string scale. Note however, that the curvaton scenario in its simplest form can only work for Hubble constants $H > 10^7$ GeV. As can be seen from figure 10, where we plotted H vs. κ for the same parameters as in figure 9, this implies $\kappa > 10^{-3} - 10^{-4}$. No such constraint exists for the inhomogeneous reheat scenario.

5.1 Brane F-inflation

In the plots discussed above we have used the GUT value for the gauge coupling $g \sim g_{\rm GUT}$ and varied the trilinear coupling κ. In brane models the gauge coupling may differ by a few orders of magnitude from the GUT value; moreover the gauge and coupling constant are no longer independent. This does not affect the inflaton contribution to the quadrupole (which can now be given as a function of g instead of κ), but it does change the string-contribution (through $\epsilon(\beta)$).

Here we discuss as an example F-term inflation that arises as a certain limit of P-term brane inflation models [7]. P-term inflationary models in $N = 1$ supergravity interpolate between F- and D-term models. The choice of a particular model is determined by the VEV of the auxiliary triplet field of P-inflation, which depends on the fluxes on the branes [7].

The matter content of P-inflation is an $N = 2$ charged hypermutiplet and a U(1) gauge multiplet. These contain in addition to the gauge bosons a pair of complex conjugate fields which can be identified with our ϕ_+ and ϕ_- fields and a neutral singlet which we denote by S. The $N = 1$ superpotential is given by

$$W = \sqrt{2} g S(\phi_+\phi_- - M^2)\,. \tag{5.1}$$

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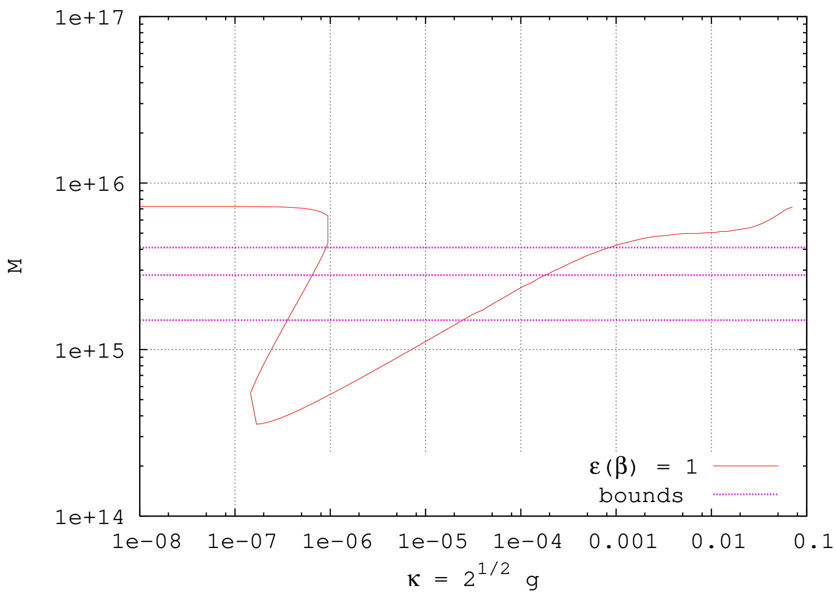

Figure 11: M vs. κ for $\mathcal{N} = 1$ and $\epsilon(\beta) = 1$ corresponding to F-term P-inflation. Also shown are, from top to bottom, the 10% bound, the JS-bound and the pulsar bound.

Hence, we recover the superpotential given in eq. (2.1) with $\kappa = \sqrt{2}g$. The strings that form at the end of F-term P-inflation have $m_A = m_\phi$, and satisfy the Bogomolny bound. Their tension is $\mu = 2\pi M^2$, i.e., $\epsilon(\beta)$ in eq. (3.1) equals unity. This in turn modifies the parameter range allowed by the data. In figure 11 we plot M versus κ for $\epsilon(\beta) = 1$. The 10%-bound, the JS-bound, and the pulsar bound on M are proportional to $\epsilon(\beta)^{-1/2}$ (see eqs. (3.6), (3.7), (3.8)), and are more stringent than for GUT F-term inflation. The JS-bound gives $10^{-6} \lesssim g \lesssim 10^{-4}$, which is much below the expected range for g.[4]

6. Warm inflation

If the inflaton, or a field coupling to the inflaton, can decay during inflation, it has a propagator of the Breit-Wigner form with an imaginary part related to the decay rate Γ. Upon calculating the one-loop correction to the inflaton potential, these contributions related to the decay rate lead to dissipative effects [45]. In the adiabatic-Markovian limit, satisfied for $\dot{\phi}/\phi < H < \Gamma$, this can be described by an effective friction term Υ in the inflaton equation of motion

$$\ddot{S} + (3H + \Upsilon)\dot{S} + \frac{\partial V}{\partial S} = 0\,. \tag{6.1}$$

In hybrid inflation inflaton decay is kinematically forbidden. However, the heaviest Higgs field $\chi_+ = (\phi_+ + \phi_-)/\sqrt{2}$ can decay into its fermionic superpartner $\tilde{\chi}_+$ and an inflatino. Since the masses of χ_+ and $\tilde{\chi}_+$ are close together, this decay is phase space suppressed.

[4]The 10% bounds implies $g < 10^{-3}$ which differs from result $g < 10^{-4}$ quoted in [19]. This difference can be traced back to the different y values used: $y = 3$ in our case and $y = 9$ in [19].

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As shown in ref. [20] its effects can be neglected during inflation. Dissipation can only be important if the inflaton sector couples to light particles. We consider the coupling

$$W = \lambda \phi_+ NN \,. \tag{6.2}$$

If hybrid inflation is embedded in a grand unified theory, and the U(1) broken at the end of inflation corresponds to baryon−lepton number, the above term naturally arises as a mass term for the right handed neutrino.

Physically, what happens is that during inflation the slowly changing inflaton field can excite the Higgs field ϕ_+, which can decay into massless N fields and its fermionic superpartners. Through this channel, the inflaton sector dissipates radiation: $\dot{\rho}_\gamma + 4H\rho_\gamma = \Upsilon \dot{S}^2$.

One can distinguish three regimes, depending on the strength of the effective damping term Υ and on the temperature $T \sim \rho_\gamma^{1/4}$ of the radiation bath.

1. $\Upsilon, T < H$

   This is the regime of cold inflation where dissipation is negligible small. The Hubble parameter sets the friction term in the inflaton equation of motion, as well as the scale of inflaton perturbations $\delta S^2 \propto H^2$.

2. $H < T < \Upsilon$

   Warm inflation in the weak dissipative regime. The friction term is dominated by the Hubble constant, i.e., by the expansion of the universe, but the perturbations are dominated by thermal effects and $\delta S^2 \propto HT$.

3. $H < \Upsilon$

   Warm inflation in the strong dissipative regime. The damping of the inflaton field is dominated by Υ, i.e., by dissipative effects. Moreover, fluctuations are thermal with $\delta S^2 \propto \sqrt{H\Upsilon}T$.

Warm inflation, which occurs for sufficiently large couplings κ, λ, lowers the scale M for a given κ compared to cold inflation. The reason is that the fluctuations δS are larger by a factor $\sqrt{T/H}(1 + (\Upsilon/H)^{1/4})$, so that a smaller M is required to obtain the right temperature anisotropy. In the strong dissipative regime there is the additional effect that the inflaton is stronger damped as compared to cold inflation, and the inflaton value when observable scales leave the horizon, S_Q, is lowered.

We will list here the important formulas governing the density perturbations; more details can be found in [20, 45]. The decay rate for the process $\phi_+ \to NN$ is $\Gamma = \lambda^2 m_+/(16\pi)$, with $m_+^2 = \kappa^2(|S|^2 + M^2)$ and λ the coupling in eq. (6.2). We can define the ratio $r \equiv \Upsilon/(3H)$ which is given by

$$r(x) = \frac{\kappa^2}{128\sqrt{3}\pi} \left(\frac{\lambda^2}{16\pi}\right) \frac{x^2}{\sqrt{1+x^2}} \frac{m_{\rm p}}{M} \,. \tag{6.3}$$

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The dissipative effects parametrized by $r(x)$ are maximized in the limit $\kappa, \lambda \to 1$ large, as this maximizes the decay rate. The formulas for the density parameters then generalize as follows. The number of e-folds is

$$N_Q = \int_{\sigma_{\rm end}}^{\sigma_Q} \frac{1}{m_{\rm p}^2} \frac{V}{V'} (1+r) {\rm d}\sigma \,. \tag{6.4}$$

The slow roll parameters change to $\epsilon_\Upsilon = \epsilon/(1+r)^2$, $\eta_\Upsilon = \eta/(1+r)^2$, with ϵ, η given in eq. (4.2). In addition a third slow roll parameter can be defined $\bar{\epsilon}_\Upsilon = \beta_\Upsilon r/(1+r)^3$ with

$$\beta_\Upsilon = m_{\rm p}^2 \frac{V'}{V} \frac{\Upsilon'}{\Upsilon} \,. \tag{6.5}$$

Inflation ends when one of the slow roll parameters exceeds unity, or when the energy inflaton decay products ρ_γ becomes larger than V_0, with

$$\frac{\rho_\gamma}{H^4} = \frac{9}{2} \frac{r\epsilon}{(1+r)^2 \kappa^2} \left(\frac{m_{\rm p}}{M} \right)^4 . \tag{6.6}$$

One can define a corresponding temperature $T \approx \rho_\gamma^{1/4}$. The density perturbations generalize to

$$\left(\frac{\delta T}{T} \right)_\phi = \frac{1}{12\sqrt{5}\pi m_{\rm p}^3} \frac{V^{3/2}}{V'} \left(1 + \sqrt{\frac{T}{H}} \right) \left(1 + \left(\frac{\pi \Upsilon}{4H} \right)^{1/4} \right) . \tag{6.7}$$

In the limit $r \to 0$ (and thus also $\Upsilon \to 0$, $T \to 0$) all above formulas reduce to those of standard cold inflation, where dissipative effects are negligible small. Finally, we give the spectral index in the various regimes

$$n_s - 1 = \begin{cases} -6\epsilon + 2\eta, & \text{for } \Upsilon, T < H \\ -\dfrac{17}{4}\epsilon + \dfrac{3}{2}\eta - \dfrac{1}{4}\beta_\Upsilon \,, & \text{for } \Upsilon < H < T \\ \left(-\dfrac{9}{4}\epsilon + \dfrac{3}{2}\eta - \dfrac{9}{4}\beta_\Upsilon \right) (1+r)^{-1} \,, & \text{for } H < \Upsilon, T \end{cases} \tag{6.8}$$

In figure 12 we show the effects of dissipation for different values of $\lambda = 1, 0.1, 0.01, 0$ and $\mathcal{N} = 1$. As expected dissipation is only important for large κ and λ, and the scale of inflation is lowered. For $\lambda \gtrsim 0.1$ all bounds are evaded. For smaller coupling the importance of the dissipative effects diminishes, e.g. for $\lambda \gtrsim 0.01$ the bound is only slightly improved. The spectral index for the same parameters is shown in figure 13. The spectrum is scale invariant over the whole κ range consistent with CMB data. For $\lambda \sim 1$ there is a discontinuity in the spectral index, which corresponds to the transition from the weak to the strong dissipative regime; it is merely an artifact of the approximation used in eq. (6.8).

Figure 14 shows the mass scale M as a function of κ, now for $\lambda = 1, 0.5, 0.1, 0$ and $\mathcal{N} = 16$. The effect of dissipation is smaller than for $\mathcal{N} = 1$. Only for larger, order one, couplings are all bounds evaded. Already for $\lambda = 0.1$ there is no enlargement of parameter space.

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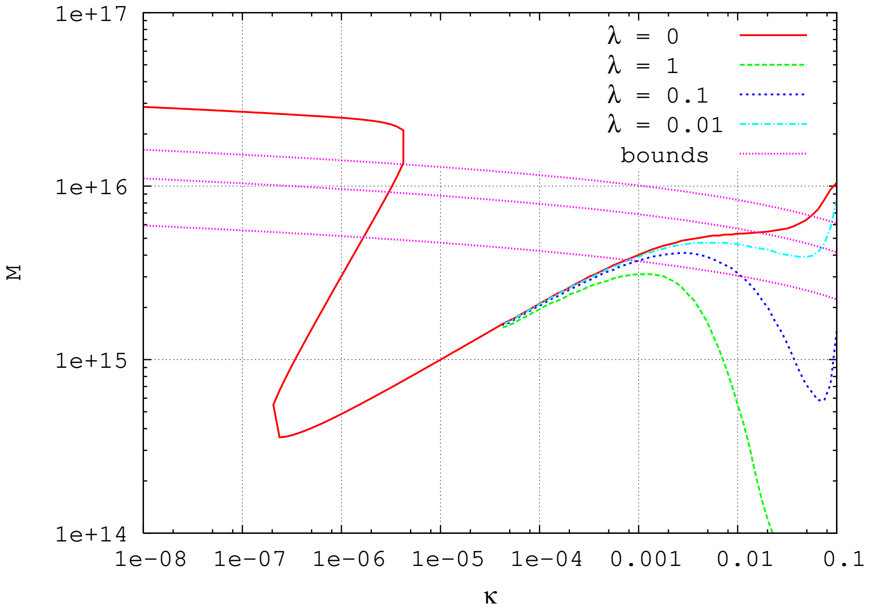

Figure 12: M vs. κ for $\lambda = 1, 0.1, 0.01, 0$ and $\mathcal{N} = 1$. Also shown are, from top to bottom, the non-gaussianity bound, the 10% bound, and the pulsar bound.

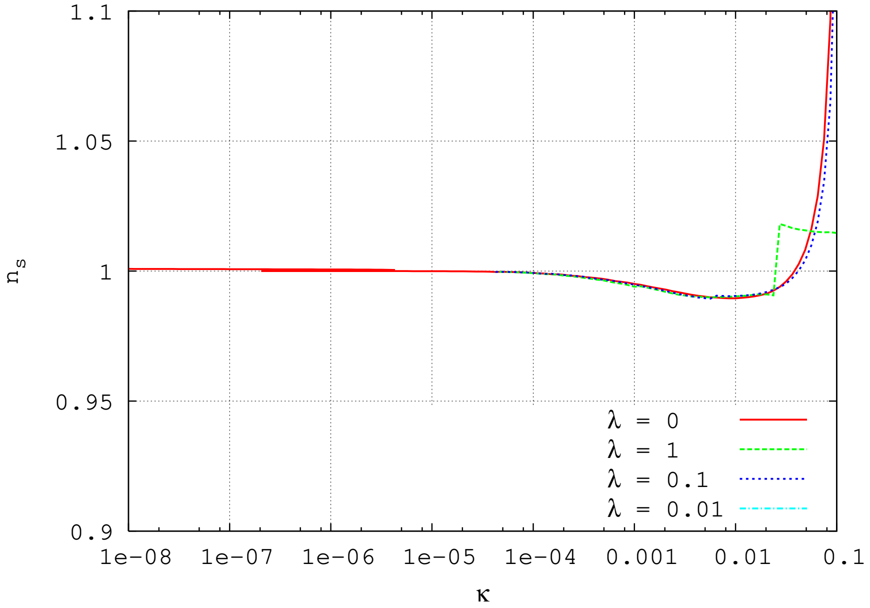

Figure 13: n vs. κ for $\lambda = 1, 0.1, 0.01, 0$and $\mathcal{N} = 1$.

Lastly, we show in figure 15 the effects of dissipation for the Bogomolny strings arising in F-term P-inflation, for which $\epsilon(\beta) = 1$. Plotted is M vs. κ for $\lambda = 1, 0.1, 0.01, 0$. Just as for GUT strings with $\mathcal{N} = 1$ are the bounds considerably improved for $\lambda < 0.1$.

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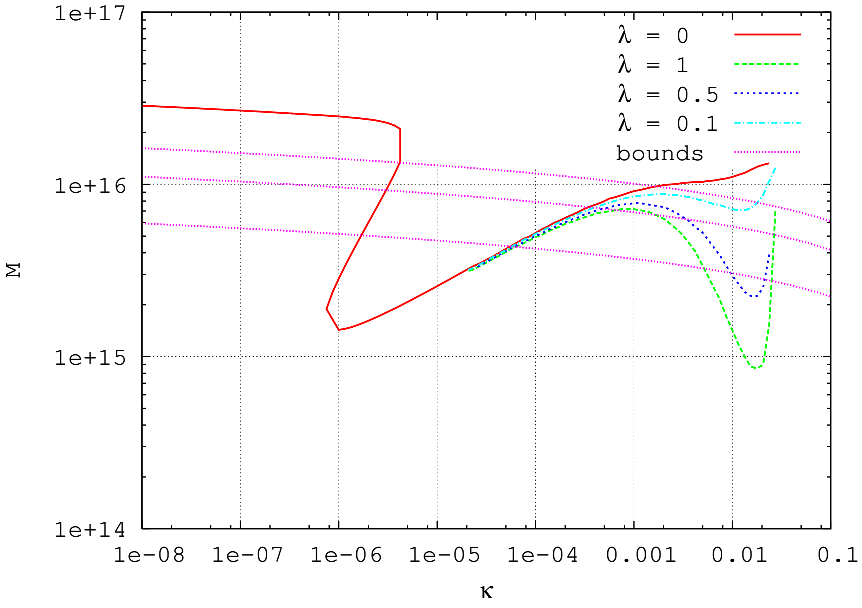

Figure 14: M vs. κ for $\lambda = 1, 0.5, 0.1, 0$ and $\mathcal{N} = 16$. Also shown are, from top to bottom, the non-gaussianity bound, the 10% bound, and the pulsar bound.

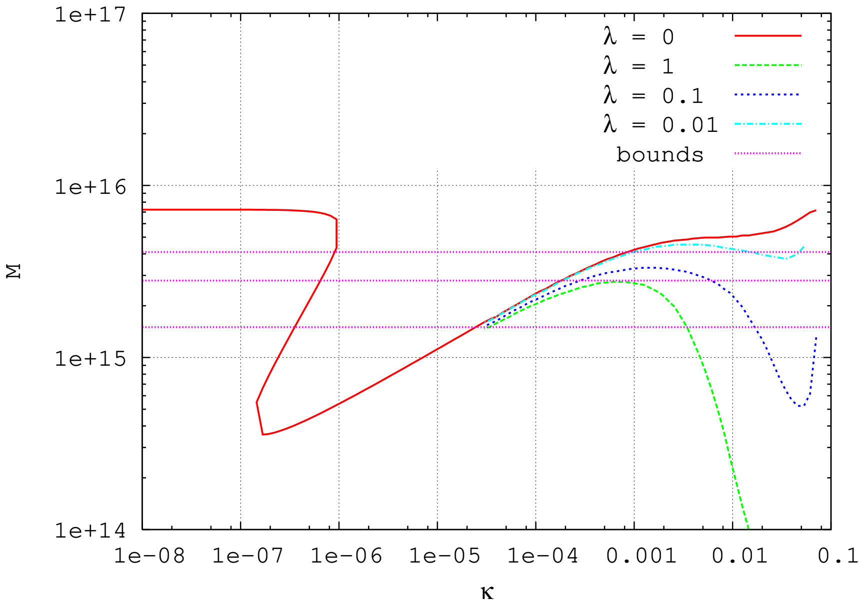

Figure 15: M vs. κ for $\lambda = 1, 0.1, 0.01, 0$ and $\mathcal{N} = 1$. Also shown are, from top to bottom, the non-gaussianity bound, the 10% bound, and the pulsar bound.

7. Conclusions

Cosmic strings form at the end of standard hybrid inflation. Both the string and inflaton contribution to the CMB are proportional to the same symmetry breaking scale M. This makes hybrid inflation testable via CMB experiments. In this paper we have determined

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the parameter space for which standard F-term hybrid inflation is compatible with the CMB data. We considered both GUT F-term inflation and P-term F-inflation.

We pointed out that cosmic strings forming at the end of GUT F-inflation are not in the Bogomolny limit. The string tension depends on the ratio of the masses of the Higgs and gauge fields, and decreases in the limit of small quartic coupling constant. This is why the bounds on M are relaxed compared to P-term models where the strings do satisfy the Bogomolny bound.

We studied the inflationary scalar potential including all possible soft and dissipative terms. Supergravity corrections are calculated assuming general expectation values in the hidden sector. The one-loop potential and the non-renormalisable terms are general, and independent of the SUSY breaking mechanism in the true vacuum. The inflaton mass and linear A-term, on the other hand, depend sensitively on the SUSY breaking mechanism. These terms are large in a canonical gravity mediated SUSY breaking scheme, in conflict with the CMB data. In order to evade the bounds, either these terms must be tuned, or low energy SUSY breaking should take place after inflation, or gauge mediation should be assumed.

Knowledge of the scalar potential of F-inflation allows for a calculation of the perturbation spectrum. We have derived analytical formulas for the symmetry breaking scale M as a function of the superpotential coupling κ in the limit that one term dominates the potential. These results agree well with our numerical calculations. The string tension, and thus the SSB scale M, is bounded by the data. We used three different bounds, the 10% bound, the Jeong-Smoot bound and the pulsar bound. For GUT strings, we find that a superpotential coupling $10^{-7}/\mathcal{N} \lesssim \kappa \lesssim 10^{-2}/\mathcal{N}$, with $\mathcal{N}$ the dimension of the Higgs-representation, is still compatible with the data.[5] The bounds are stronger for the strings formed in P-inflation: $10^{-7} < \kappa < 10^{-4}$.

Finally we considered ways to ameliorate the CMB bounds. In the curvaton or inhomogeneous reheat scenario not the inflaton but some other scalar field is responsible for the density perturbations. Lowering the contribution of the inflaton to the CMB, even by only 10%, immediately evades all bounds. Warm inflation can occur if the Higgs field is coupled to light fields with a large, order one, coupling. This opens up parameter space for large superpotential couplings.

Acknowledgments

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[5] We note that although ref. [19] considers GUT inflation, they do not take the corrections away from the Bogomolny limit into account, and consequently they find a much stronger bound.

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